Base 2? Where Did it Come From?

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February 2007

References:

Rabdology by John Napier. Translated by William Frank Richardson. No. 15 in the Charles Babbage Institute Reprint Series for the History of Computing. Published in 1990 by the Massachusetts Institute of Technology and Tomash Publishers. (originally appeared in Latin in 1617)

History of Binary and Other Nondecimal Numeration by Anton Glaser. Published in 1971 by Anton Glaser.

A) Western tribes of Torres Straights:

1 → *urapun*

2 → *okosa*

3 → *okosa urapun*

4 → *okosa okosa*

5 → *okosa okosa urapun*

6 → *okosa okosa okosa*

More than 6 → *ras*

The arithmetic of these tribes could be considered as a base two arithmetic.

B) Egyptian Algorithm (Duplication Algorithm)

19 x 35 =

→	1 x 35 =	35
→	2 x 35 =	70
	4 x 35 =	140
	8 x 35 =	280
→	16 x 35 =	560

We know 19 = 1 + 2 + 16 = $2^0 + 2^1 + 2^4$

Add the numbers in the right column on the same rows as 1, 2, and 16 times 35:: 35 + 70 + 560 + 665. 665 is the desired product.

C) Tower of Hanoi

The puzzle called a *Tower of Hanoi* was invented by Edouard Lucas in 1883.

It consists of 3 pegs, a number, n, of disks (all different diameters). Stack all the disks on one peg, largest disk on the bottom, and so on.

Object is to transfer the ”tower” to another peg, and determine the minimum number of moves to make the transfer.

D) Computers: John von Neumann, 1945 white paper

Contained in the *First Draft of a Report on the EDVAC* (Electronic Discrete Variable Automatic Computer), 1945:

"… Thus, whether the tubes are used as gates or as trigger, the all-or-none, two equilibrium arrangements are the simplest ones. Since these tube arrangements are the handle numbers by means of their digits, it is natural to use a system of arithmetic in which the digits are also two valued. This suggests the use of the binary system."

E) Leibniz

First published paper with binary numbers, 1703
(0 denoting nothing and 1 denoting God)

1703 paper: *An Explanation of Binary Arithmetic Using only the Characters 0 and 1, with Remarks about it's Utility and the Meaning it Gives to the Ancient Chinese Figures of Fuxi.* (More about this paper can be found in the Anton Glaser reference given at the top of this paper)

In this paper Leibniz writes about binary numeration:

"it permits new discoveries [in]…arithmetic...in geometry, because when the numbers are reduced to the simplest principles, like 0 and 1, a wonderful order appears everywhere…

The binary calculations "… are so easy that we shall never have to guess or apply trial and error, as we must do in ordinary division. Nor do we need to learn anything by rote…"

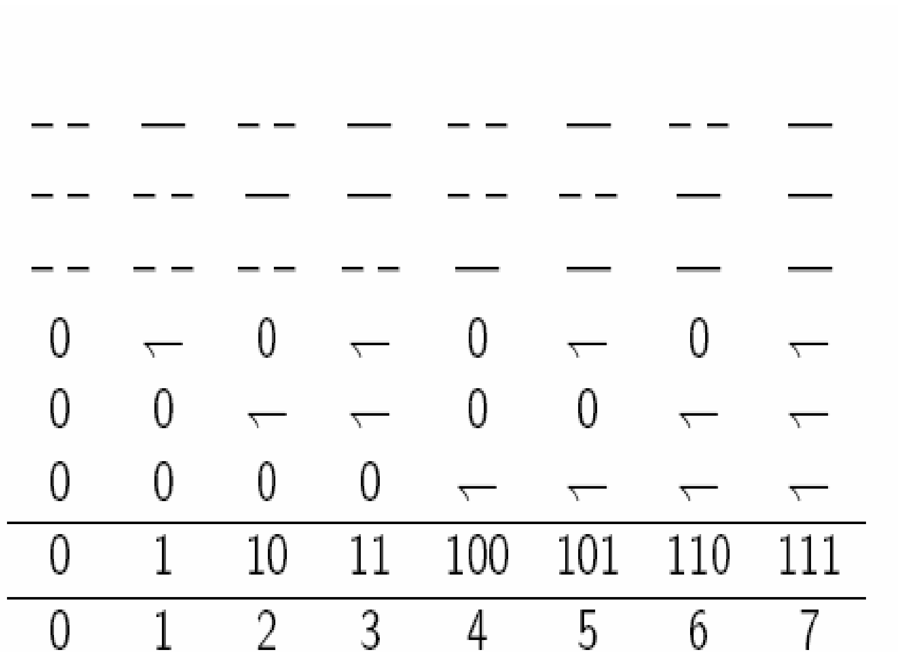
Leibniz found many patterns in binary numbers, such as can be seen in the table below:

Table of Numbers

○	○	○	○	○	0	0
○	○	○	○	○	1	1
○	○	○	○	1	0	2
○	○	○	○	1	1	3
○	○	○	○	0	0	4
○	○	○	1	0	1	5
○	○	○	1	1	0	6
○	○	○	1	1	1	7
○	○	1	0	0	0	8
○	○	1	0	0	1	9
○	○	1	0	1	0	10
○	○	1	0	1	1	11
○	○	1	1	0	0	12
○	○	1	1	0	1	13
○	○	1	1	1	0	14
○	○	1	1	1	1	15
○	1	0	0	0	0	16
○	1	0	0	0	1	17
○	1	0	0	1	0	18
○	1	0	0	1	1	19
○	1	0	1	0	0	20
○	1	0	1	0	1	21
○	1	0	1	1	0	22
○	1	0	1	1	1	23
○	1	1	0	0	0	24
○	1	1	0	0	1	25
○	1	1	0	1	0	26
○	1	1	0	1	1	27
○	1	1	1	0	0	28
○	1	1	1	0	1	29
○	1	1	1	1	0	30
○	1	1	1	1	1	31
1	0	0	0	0	0	32

etc.

Chinese connection: Leibniz corresponded with the Reverend Father Bouvet, a French Jesuit living in Peking. Leibniz saw a connection between the symbols found in an ancient Chinese work (abut 4,000 years old), I-Ching and thought it might b the origin of a universal symbolic language.



F) In 1617 John Napier published *Rabdology* , the book where he introduced his “Bones” or “Rods” to the world. In the last section of this book, Section III, was called ”Location Arithmetic as Performed on a Chessboard”. In reality this section talks about binary numbers, except rather than using 1’s and 0’s Napier used letters of the alphabet. In doing so, this was no longer a place value system, but a additive number system

In this system a = 1 = 2^0 , b = 2 = 2^1 , c = 4 = 2^2 , etc. The following table (or rod as Napier would say) was used to change “ordinary numerals” (our Hindu-Arabic numerals) to the “Location Numerals”.

First way to convert 1611 to a location numeral is by using subtraction. The 1611 is placed in the row with the 1024 since 1611 is between 1024 and 2048. Then 1024 is subtracted from 1611. The result is placed in the row with the 64 – etc.

2048	m			
1024	l	1611	l	1611-1024 =587
512	k	587	k	587-512 = 75
256	i			
128	h			
64	g	75	g	75-64=11
32	f			
16	e			
8	d	11	d	11-8 = 3
4	c			
2	b	3	b	3-2 = 1
1	a	1	a	

So 1611 → lkgdba , and the letters can be written in any order.

The second way to convert 1611 to a location number is by "halving" . This time the 1611 is placed in the row with the 1. If the number is odd, and 1611 is odd, then 1 is subtracted from 1i611, and half of that number is placed one row up. When the number is odd the letter from that row is used in the location number.

2048	m			
1024	l	1	l	odd
512	k	3	k	odd
256	i	6		even
128	h	12		even
64	g	7525	g	odd
32	f	50		even
16	e	100		even
8	d	11201	d	odd
4	c	402		even
2	b	805	b	odd
1	a	166	a	even

And again 1611 → lkgdba

To add location numbers you use ***Abbreviation*** – two counters in a certain position are to be replaced by one in the next higher position.

For example: $abbdeefg \rightarrow acdffg \rightarrow acdgg \rightarrow acdh$

To subtract location numbers you use ***Extension*** - means that a single counter in a certain position is replaced by two in the next lower position.

For example: $acdeh \rightarrow acdegg \rightarrow acdeffg \rightarrow$
$acdeeefg \rightarrow acdddeefg \rightarrow$
$acccddeefg \rightarrow abbccddeefg$

Neither Abbreviation nor Extension alters the value of the number.

For the following two numbers, we have

$90 = 2 + 8 + 16 + 64 \rightarrow bdeg$

$51 = 1 + 2 + 16 + 32 \rightarrow abef$

Adding:

$$\begin{aligned} bdeg + abef &= bdegabef \\ &= abbdeefg \\ &= acdffg \\ &= acdgg \\ &= acdh \end{aligned}$$

Subtractracting:

$$\begin{aligned} bdeg - abef &= bdeff = abef \\ &= df - a \\ &= ccf - a \\ &= bbcf - a \\ &= aabcf - a \\ &= abcf \end{aligned}$$

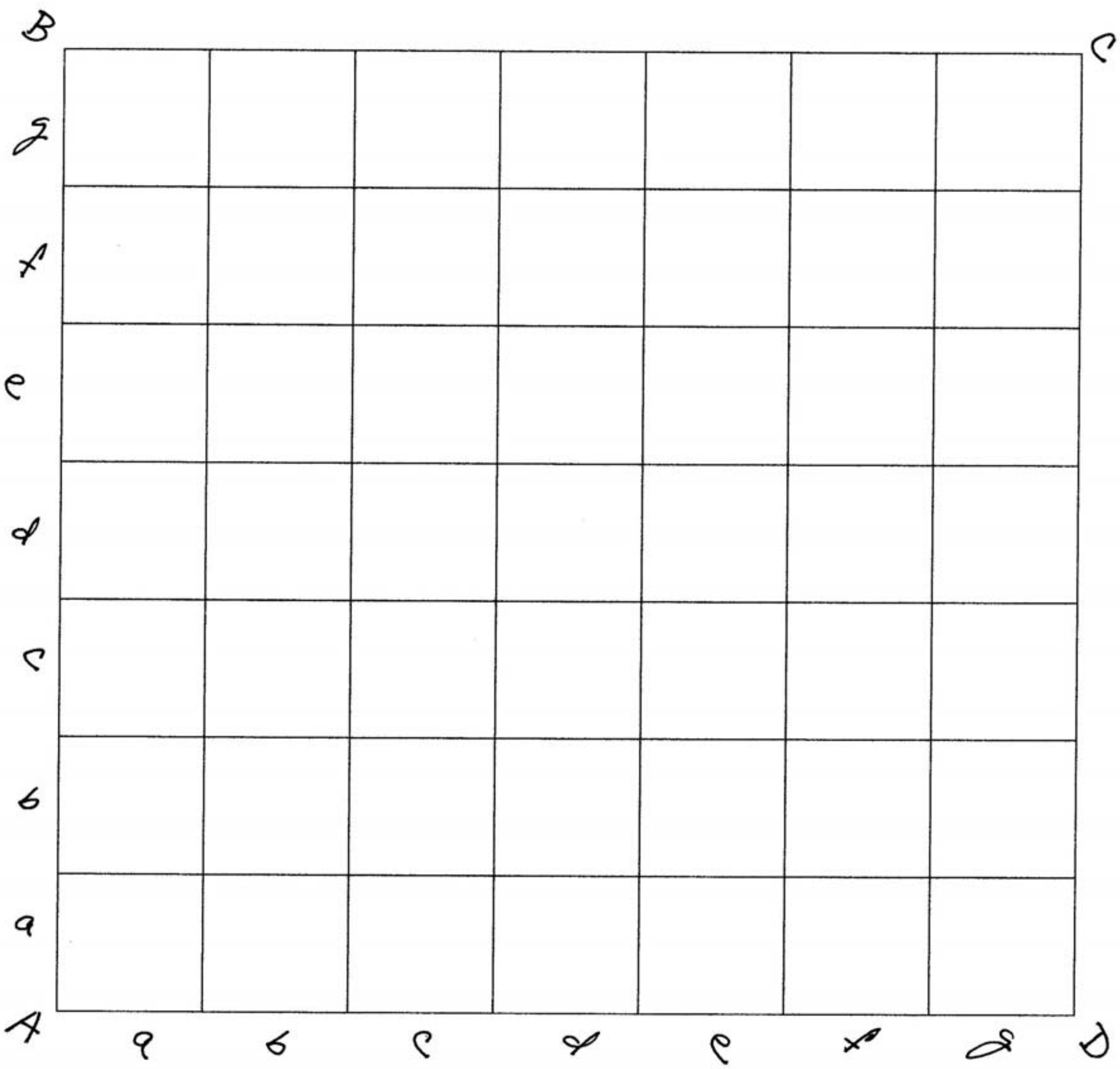
Multiplication is "easier" if you use placements in two dimensions, as on a chessboard, than using the one dimensional rod.

There are two kinds of motion on the chessboard:

<u>Direct motion</u> is parallel to the sides – parallel to AB, DC, AD, or BC. In direct motion the value of each space is double that of the one before.

<u>Diagonal motion</u> proceeds from one corner of the board to the diagonally opposite corner, or is parallel to this motion. All squares lying diagonally between two identical letters have the same value as the number noted in either margin.

Board for Multiplication Using Location Numerals



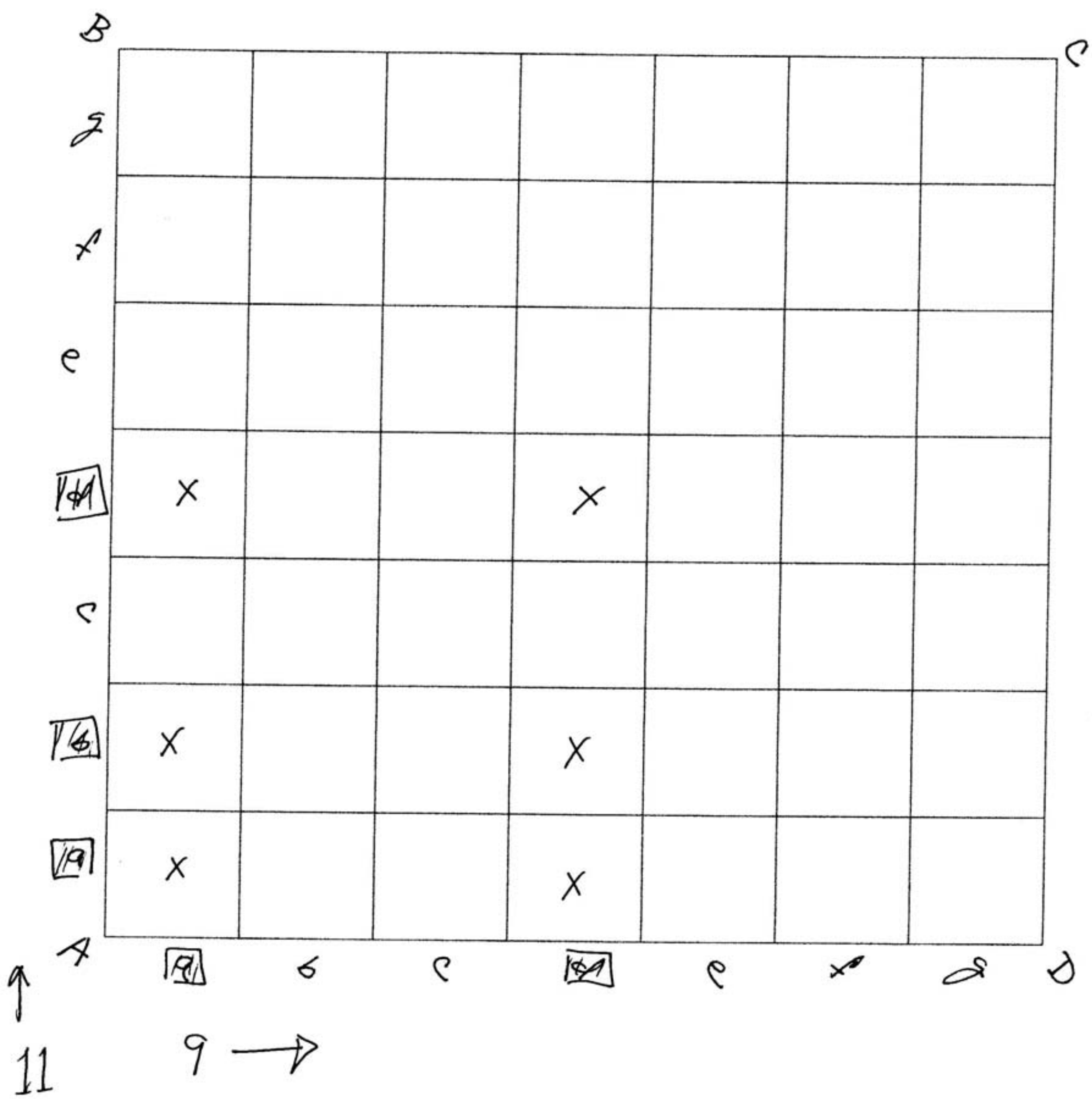
To multiply 11 x 9 = 99 on the “chessboard”, first find the location numerals for 11 and 9.

11 = 1 + 2 + 8 → *abd*

9 = 1 + 8 → *ad*

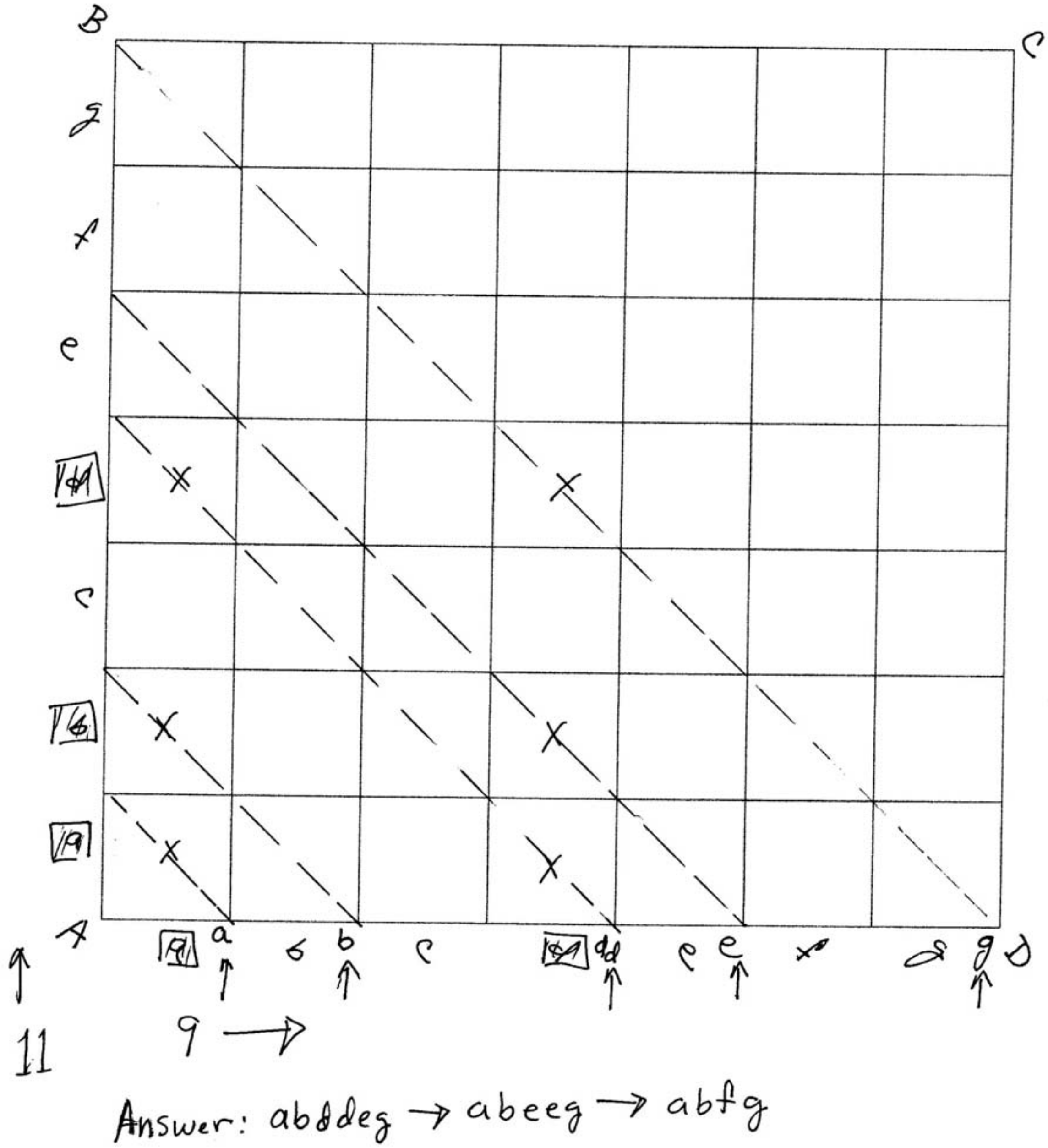
Place them on the edges of the board, and mark where the horizontal and vertical rows intersect.

Board for Multiplication Using Location Numerals



Now draw diagonal lines and read the answer:

Board for Multiplication Using Location Numerals



Note that: 99 = 1 + 2 + 32 + 64 → abfg

Napier’s Location Numerals could be an interesting way to transition from additive numeral systems to place value systems. It could also be fun to explore division using this system.